NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 7
COURSE CODE: RAN701S	**COURSE NAME:** REAL ANALYSIS
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	PROF. G. HEIMBECK
MODERATOR:	PROF. F. MASSAMBA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

Question 1 [15 marks]

Let X be a set of real numbers.

a) What is an upper bound of X? State the definition. [3]

b) Let $u \in \mathbb{R}$ be an upper bound of X and $v \in \mathbb{R}$ such that $u \leq v$. Show that v is an upper bound of X. [5]

c) Let $U(X)$ denote the set of all upper bound of X. Is $U(X)$ bounded above? Substantiate your answer. [7]

Question 2 [15 marks]

Let X be an infinite set of natural numbers. Consider $\pi : \mathbb{N} \to X$ with

$$\pi(1) = \min X, \qquad \pi(n+1) = \min(X - \{\pi(1), \pi(2), \ldots, \pi(n)\}) \text{ for all } n \in \mathbb{N}.$$

a) How does one call this description of π? Show that it does make sense. [5]

b) Prove that π is strictly increasing. [7]

c) Is π injective? Explain. [3]

Question 3 [14 marks]

Consider the following sequence

$$b_n := \left(1 + \frac{1}{n}\right)^{n+1} \text{ for all } n \in \mathbb{N}.$$

a) Prove that this sequence is strictly decreasing. [7]

b) Is this sequence convergent? If it is convergent, determine its limit. [5]

c) Does this sequence have divergent subsequences? Explain. [2]

Question 4 [13 marks]

a) What is a series of real numbers? Explain this concept. [3]

b) Let $\sum a_k$ and $\sum b_k$ be series of real numbers. The summation starts at 1.

i) Prove that

$$\sum_k a_k + \sum_k b_k = \sum_k (a_k + b_k).$$

[5]

ii) If $\sum a_k$ and $\sum b_k$ are convergent, show that $\sum (a_k + b_k)$ is convergent and

$$\sum_{k=1}^{\infty} (a_k + b_k) = \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k.$$

[5]

Question 5 [15 marks]

Let $X \subset \mathbb{R}$.

a) What is an accumulation point of X? State the definition. [3]

b) Let $a \in \mathbb{R}$ be an accumulation point of X such that $a \notin X$. Prove that there exists a sequence in X which converges to a. What does one need to prove this? State the fact. [7]

c) Show that every accumulation point of X belongs to the closure $\overline{X}$ of X. [5]

Question 6 [14 marks]

Let $p: \mathbb{R} \to \mathbb{R}$ be defined by $p(x) := x^7 + x + 1$.

a) Show that p is strictly increasing. [3]

b) Prove that p is surjective. [6]

c) Show that p has exactly one zero. Verify that the zero of p is a number between -1 and 0. [5]

Question 7 [14 marks]

a) Let $a, b \in \mathbb{R}$ such that $a < b$ and let $f: [a, b] \to \mathbb{R}$ be a function which is differentiable on (a, b) and continuous at a and b.

   i) What is the domain of the derivative f' of f? Explain. [5]

   ii) If $f' = 0$, prove that f is constant. [5]

b) If the derivative of a function is the zero function, is the function constant? Either prove that the function is constant or make a counterexample. [4]

End of the question paper